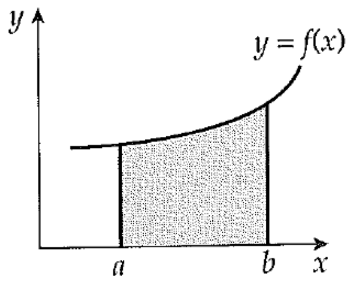
6) Areas under curves

The definite integral $\int_a^b f(x)\ dx$ gives the area under the curve $f(x)$ for the interval $a \leq x \leq b$.



The integral sign $\int$ means 'the sum of', and the process adds all the areas of very tiny width between the limits of the integration. The limits in the diagram above are a and b.

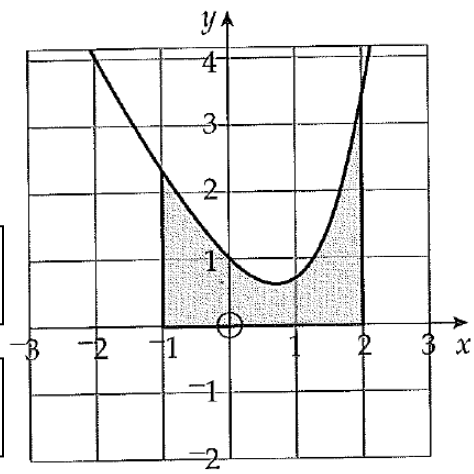
Later we will look at ***numerical*** methods of adding all the areas of very tiny width to find the total area under the curve, but for now we will use means of integration to find it.

Example 1: Calculate the area under the curve
$y = e^x - 2x$ between the values
$x = -1$ and $x = 2$.

Area = $\int_{-1}^{2} e^x - 2x\ dx$
$= [e^x - x^2]_{-1}^{2}$
$= [e^2 - 2^2] - [e^{-1} - (-1)^2]$
= 4.02 units (2dp)

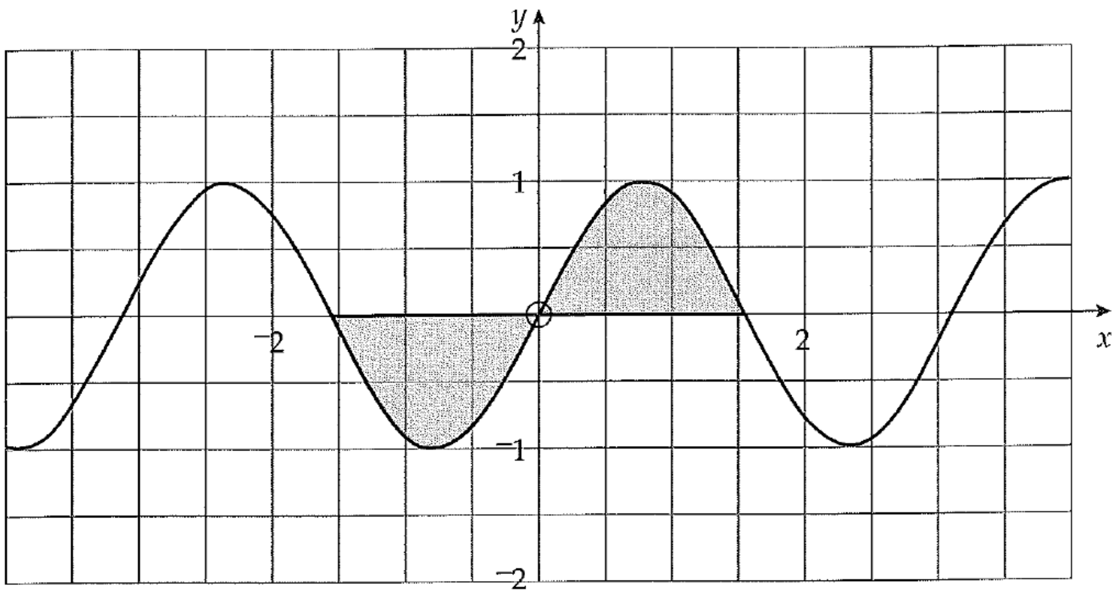
must show the integrated fn in square brackets

won't need this line of working, use GC



If the area we are looking at has parts of the graph sitting below the x-axis, we must separate the area into parts, because the area under the x-axis will generate a negative answer and result in an incorrect value.

Example 2: Find the shaded area for the curve $y = \sin(2x)$. Notice the symmetry on the shape. You are given that the x-intercepts are: $\frac{-\pi}{2},\ 0,\ \frac{\pi}{2}$.



Split the graph into two parts, one interval is $\frac{-\pi}{2} \le x \le 0$ (the part of the graph that is under the x-axis) and the second interval is $0 \le x \le \frac{\pi}{2}$ (the part of the graph is that above the x-axis). We will take the absolute value of the integral of the first interval, as it will come up negative.

Area = $\left|\int_{-\pi/2}^{0} \sin(2x)\ dx\right| + \int_{0}^{\pi/2} \sin(2x)\ dx$

$= \left|\left[\frac{-1}{2}\cos(2x)\right]_{-\pi/2}^{0}\right| + \left[\frac{-1}{2}\cos(2x)\right]_{0}^{\pi/2}$

$= \left|\ \left[\frac{-1}{2}\cos(2(0)\right] - \left[\frac{-1}{2}\cos(2(\frac{-\pi}{2})\right]\ \right| + \left[\frac{-1}{2}\cos(2(0)\right] - \left[\frac{-1}{2}\cos(2(\frac{\pi}{2})\right]$

$= |-1| + 1$

$= 1 + 1$

= 2 units^2

Delta Ex 20.1, pg 186 – 187
Ex 20.2, pg 190